A METHOD OF DETERMINING DIFFUSION COEFFICIENTS AND FRICTIONAL DRAG FACTORS IN LIQUIDS

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We propose a method of modeling the hydrodynamic friction of plates and the molecular diffusion of an impurity in a liquid medium by means of the phenomena of convection mass transport.

Considerable interest has recently been expressed in the problem of the resistance and heat and mass transfer of liquid media in drop form, primarily, in non-Newtonian solutions, melts, suspensions, emulsions, etc.

Within the framework of the theory of incompressible continuous media, the mathematical descriptions of isothermal motions of incompressible gases and liquids are identical. Consequently, we ignore the specific differences in the mechanical behavior of gases and liquids. In particular, it is well known that the aeromechanical measurements of velocity and pressure are considerably better and more reliable than, for example, the hydromechanical methods. Experiments with non-Newtonian fluids are particularly difficult. Experimental data on the resistance of such fluids are very hard to find.

Analysis of literature data shows that the information obtained with various pneumometric nozzles, with static pressure probes at the surface being streamlined, with heated thermoanemometric sensors, etc.,

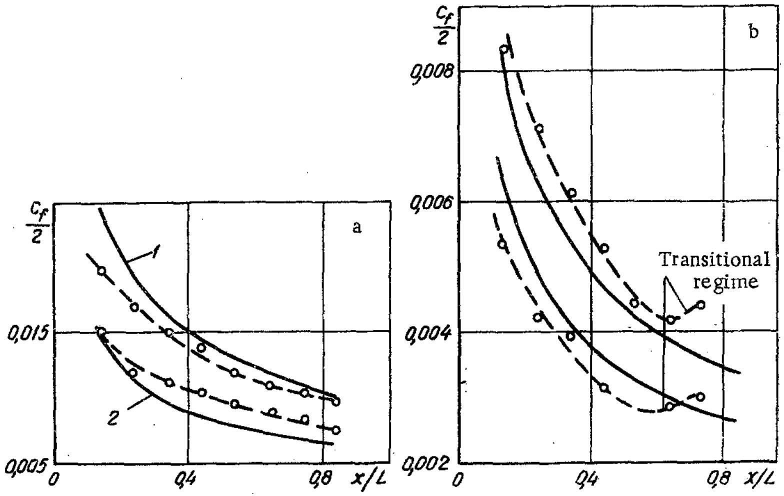

Fig. 1. Comparison of calculations (self-similar exact solutions: solid curves) with the approximate values (dots) of the electrochemoluminescence experiments: a) aqueous solution of carboxymethylcellulose ($n = 0.88$; $c = 1\%$); 1) $U_\infty = 0.21$ cm/sec; 2) 50 cm/sec; b) Newtonian solution without polymer additions ($n = 1.0$).

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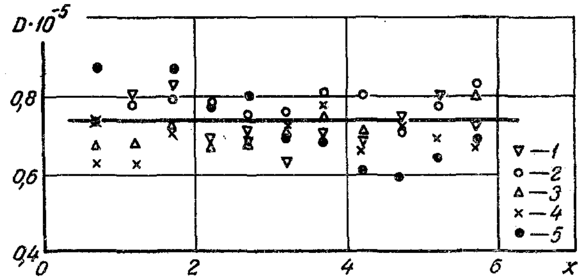

Fig. 2. Values of the diffusion coefficients for H_2O_2 ions in Newtonian electrolysis on the basis of measurements of the local j-flows at various points on a longitudinally streamlined plate: 1) 0.17 m/sec; 2) 0.21 m/sec; 3) 0.35 m/sec; 4) 0.45 m/sec; 5) 0.5 m/sec.

is quite limited and unreliable for liquid media. Thus, for example, it is demonstrated in the work by Barenblatt and his students [1] that such pneumometric nozzles as Pitot tubes in polymer solutions reveal a markedly nonlinear time dependence on flow velocity and on the diameter of the inlet orifice.

Most of the hydrodynamic and heat- and mass-transfer experimental research on liquid media is currently reduced to measuring the integral balances of either the entire streamlined surface, or of individual portions of that surface. At the same time, of greatest interest are the local characteristics (friction, heat and mass flows) of transfer and the most important details of the hydrodynamic situation (separation of the boundary layer, transition from laminar to turbulent regimes, the wake region, the wake itself, vortex formations, cavities, etc.).

1. The Reynolds analogy methods to find the local friction or physical modeling for viscous liquid media are not very effective, since the Prandtl numbers (heat- and mass-transfer) are, first of all, usually very much greater than unity, and secondly, as a rule, they differ markedly in terms of magnitude. Thus, for example, for aqueous solutions of the sodium salt of carboxymethylcellulose $\mathrm{Pr_t} = \tau_{\mathrm{eff}}/a$ is of the order of 10^2, whereas the order of $\mathrm{Pr_m} = \nu_{\mathrm{eff}}/D$ is approximately 10^4. Thus, even for the case of laminar streamlining of a plate, there is no similarity between the dimensionless velocity, temperature, and concentration fields.

Nevertheless, it remains possible to establish a balance between the diffusion flows and the frictional drag in the boundary layer. For this it is sufficient to have some completely reliable solution (exact or approximate) of the diffusion problem, in conjunction with the measurements of the j-flows from the experiment on convection mass transfer. The solution of the linear (or quasi-linear) problem of the diffusion boundary layer is much simpler than the integration of the nonlinear equations of the dynamic boundary layer. On the other hand, [2, 3] discuss a new and quite simple method of measuring j-flows for bodies of arbitrary configuration by means of an electrochemoluminescence procedure. Measurements of this kind combine the clarity of visualization with reliable quantitative estimates; they are distinguished by simplicity, high accuracy, and good reproducibility. On the basis of this seemingly rather limited information about local j-flows, it is nevertheless possible to derive an acceptable approximate relationship for frictional drag in liquids with any viscosity law.

As an example, let us consider the problem of a flat plate longitudinally streamlined by a non-Newtonian fluid of the "power" type. The convection diffusion calculated in [2] yields the following for the local values of the diffusion Nusselt number:

$$\mathrm{Nu}_{mx} = \frac{j_{(x)}\,x}{Dc_0} = \frac{\mathrm{Pe}_{mx}^{1/3}\,\mathrm{Re}_x^{1/3(1+n)}\,\beta_1^{1/3}}{\frac{1}{3}\Gamma\left(\frac{1}{3}\right)\sqrt[3]{18}\,[n(1+n)]^{\frac{1}{3(1+n)}}\left[1-\left(\frac{h_0}{x}\right)^{\frac{2n+1}{2(n+1)}}\right]^{1/3}}$$

$$= \frac{\mathrm{Pr}_{mx}^{1/3}\,\mathrm{Re}_x^{1/(1+n)}\beta_1^{1/3}}{\frac{1}{3}\Gamma\left(\frac{1}{3}\right)\sqrt[3]{18}\,[n(n+1)]^{\frac{1}{3(1+n)}}\left[1-\left(\frac{h_0}{x}\right)^{\frac{2n+1}{2(n+1)}}\right]^{1/3}}. \qquad (1)$$

The form parameter $\beta_1 = f''(0)$ of the dynamic field is proportional to the velocity gradient at the wall. It can be expressed [4] in terms of the frictional drag factor

$$c_f = \frac{2\tau_w}{\rho U_\infty^2} = \frac{2k\left(\frac{\partial u}{\partial y}\right)^n}{\rho U_\infty^2} = 2\,[n(1+n)]^{-\frac{n}{1+n}}\,\mathrm{Re}_x^{-\frac{1}{1+n}}\,[f''(0)]^n \qquad (2)$$

or

$$\beta_1 = \left(\frac{c_f}{2}\right)^{\frac{1}{n}}[n(1+n)]^{\frac{1}{n+1}}\,\mathrm{Re}_x^{\frac{1}{n(1+n)}}. \qquad (3)$$

Simultaneous solution of Eqs. (1) and (3) yields the unknown relationship

$$\frac{c_f}{2}=\left[\frac{1}{3}\Gamma\left(\frac{1}{3}\right)18^{1/3}\right]^{3n}\left[1-\left(\frac{h_0}{x}\right)^{\frac{2n+1}{2(n+1)}}\right]^{n}\left(\frac{n+1}{2n+1}\right)\frac{\mathrm{Nu}_{mx}^{3n}}{\mathrm{Pe}_{mx}^{n}\mathrm{Re}_x} \tag{4}$$

or

$$\frac{c_f}{2}=\left[\frac{1}{3}\Gamma\left(\frac{1}{3}\right)\left(18\frac{n+1}{2n+1}\right)^{1/3}\right]^{3n}\left[1-\left(\frac{h_0}{x}\right)^{\frac{2n+1}{2(n+1)}}\right]^{n}\frac{\mathrm{Nu}_{mx}^{3n}}{\mathrm{Pr}_{mx}^{n}\mathrm{Re}_x^{\frac{3n+1}{n+1}}}. \tag{5}$$

Finally, we can use the mass-transfer Stanton number

$$\mathrm{St}_m=\frac{\mathrm{Nu}_{mx}}{\mathrm{Pe}_{mx}}=\frac{j(x)}{c_0U_\infty}. \tag{6}$$

Then

$$\frac{c_f}{2}=\left[\frac{1}{3}\Gamma\left(\frac{1}{3}\right)\right]^{3n}\left\{18\frac{n+1}{2n+1}\left[1-\left(\frac{h_0}{x}\right)^{\frac{2n+1}{2(n+1)}}\right]\right\}^{n}\mathrm{St}_{mx}^{3n}\,\mathrm{Pr}_{mx}^{2n}\mathrm{Re}_x^{\frac{3n-1}{n+1}}. \tag{7}$$

Figure 1 shows the results from calculations on the basis of formulas (4)-(6) on the basis of experimental data taken from [2] for n = 1 and 0.83. Agreement between the approximate and exact calculation is quite satisfactory. The slight divergence can be ascribed to the elevated level of perturbations in the flow impinging on the plate, since in these experiments the plate was swept by its own weight, which had not yet settled down completely.

2. For liquids exhibiting no marked concentration relationship for the coefficient D of translational diffusion, the use of the electrochemoluminescence method is particularly effective. The most popular methods of determining D are based on optical procedures (interference, polarization, refraction, etc.) which require expensive equipment which is highly specialized and hard to obtain, since it is both complex and not in regular production. The D measurements by the electrochemoluminescence method should best be based on the data from experiments with a longitudinally streamlined plate. Then, for fixed values of c_0, k, n, ρ, and U_∞, from (1), we obtain

$$D=\sqrt{\left[\frac{j\Gamma\left(\frac{4}{3}\right)}{c_0}\right]^{3}\frac{18(n+1)}{2n+1}\left[\frac{n(n+1)k/\rho}{U_\infty^{2-n}}\right]^{\frac{1}{1+n}}\frac{x^{\frac{n+2}{n+1}}}{\beta_1U_\infty}}. \tag{8}$$

For example, in the case of a Newtonian system (n = 1) Eq. (8) is transformed [5] to the familiar Levich formula

$$j(x)\cong 0.34\frac{Dc_0\sqrt{U}}{\sqrt{\nu x}}\left(\frac{\nu}{D}\right)^{1/3}. \tag{9}$$

For any two abscissa values of x (increasing distance from the leading edge of the plate) we have

$$D_i=D_j\frac{I_i\sqrt{x_i}}{I_j\sqrt{x_j}}. \tag{10}$$

Here I is the local value of the photocurrent at the output of the photomultiplier. Thus it is possible to achieve the necessary number of steps from a brief experiment to ensure reliability and sufficient accuracy in the determination of the quantity D. The reference value of D_j is calculated from (8). The convenience of (10) rests in the fact that it, unlike (9), contains only two directly measurable characteristics – the abscissa value of x and the photocurrent I. Figure 2 shows a rather clear representation of the correlation between the values of D obtained by the electrochemoluminescence method and by the Lamm refractometric method [6] for an $H_2O-H_2O_2$ system (impurity).

The determination of D could not necessarily be associated with the information provided by the optical channel, i.e., with the measurement of the intensity of the electrochemoluminescence. The method can be simplified substantially, retaining only the electrochemical channel, using a microammeter to record the current in the outside circuit of the cell. In this case the instrument will show the integral magnitude of the current I_c over the entire anode surface and the formula for the calculation of D will assume the form [5]

$$D = \sqrt{\left(\frac{I_c}{0.68c_0b}\right)^3 \frac{1}{\nu}\left(\frac{Uh}{\nu}\right)^{-\frac{3}{2}}}\,. \tag{11}$$

To come up with the required number of independent measurements, we can vary the velocity of the flow impinging on the anode, in which case (n = 1)

$$D_i = D_j\,\frac{j_i\sqrt{U_j}}{j_j\sqrt{U_i}}\,. \tag{12}$$

This method is no less effective in determining the concentration function D(c). Many problems of nonlinear diffusion are presently being solved successfully [7, 8] for various types of D(c) functions. If the nature of the functional relationship D(c) is a priori unknown, the derivation of the required formula for the diffusion flow at the wall is achieved by means of a power expansion of D(c).

NOTATION

ν, a, and D	are, respectively, the coefficients of kinematic viscosity, thermal diffusivity, and diffusion;
j	is the local diffusion flow;
k and n	are the parameters of the exponential rheological equation;
U_∞	is the flow velocity at the outside edge of the boundary layer;
c_0	is the concentration of the active electrolyte in the volume phase;
x and y	are the longitudinal and transverse coordinates of the boundary layer;
h_0	is the length of the passive preconnected section of the plate;
h	is the overall plate length;
b	is the plate width;
τ_w	is the frictional resistance of the local plate-surface element;
f	is a dimensionless stream function (a self-similar variable);
$Pe_{mx} = U_\infty x/D$	is the mass-transfer Peclet parameter;
$Re_{mx} = U_\infty^{2-n}x^n/(k/\rho)$	is the generalized Reynolds number.

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